Applying Decision Analysis to Real Problems

DEIO

Handling Uncertainty:
Probabilities, Sensitivity Analysis, Scenarios

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Objectives and Agenda

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- Illustrate key issues in handling uncertainties in Decision Analysis
  - Subjective probabilities
  - Probabilistic influence diagrams
  - Eliciting probabilities
  - Psychological issues
  - Updating probabilities
  - A real case
  - Sensitivity Analysis
  - Scenarios

Handling uncertainty

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We shall deal with issues in relation with modelling uncertainty. We assume a certain background typical of courses in Statistics. We shall focus on topics relevant for Decision Analysis, adopted from Bayesian Inference.

An intro is in

http://en.wikipedia.org/wiki/Bayesian_inference

And in the paper Bayesian methods in conservation biology (first 4 pages)

Uncertainty

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- Uncertainty is the lack of knowledge of what is or will happen. It is almost ubiquitous in our lives. Consider these statements:
  - Smoking causes cancer
  - Madrid will organise the Olympic Games in 2020
  - Mexico became independent in 1826
  - My weight is more than 84 kgs
  - Hannibal crossed the Alps through San Bernardo
- Due to uncertainty, you do not know the consequences of your decisions before making them. Consider these examples:
  - The price of a Brent barrel will exceed 100 dollars by the end of year 2011.
  - Spain will default before the end of 2011.
  - Water demand in Madrid by 2020 will be higher than...
  - No storm is expected around the coast of Ferrol in the next two days.

Uncertainty

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- We shall use probabilities to quantify uncertainty. The three main interpretations are:

  - *Classical.* With equally likely cases, the probability of an event is defined through

    Favourable cases/Possible cases

    http://en.wikipedia.org/wiki/Classical_definition_of_probability
  - *Frequentist.* If we may repeat indefinitely an experiment under identical conditions, the probability of an event is

    Limit of relative frequency of event

    http://en.wikipedia.org/wiki/Frequency_probability

    Operationally, relative frequency in a large number of trials

Uncertainty

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- ***Subjective.*** The probability of an event is

  A measure of the degree of belief in the occurrence of the event

  http://en.wikipedia.org/wiki/Bayesian_probability

  The most general and useful concept in Decision Analysis

Uncertainty

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In general, when modeling uncertainty in a decision making problem, we need to deal with these issues:

- Which are the key uncertainties?
- Which are the possible outcomes of such uncertainties?
- Which are the probabilities of various outcomes?
- Which are the consequences entailed by such outcomes (for various alternatives)?

We deal with them in the next slides

Probabilistic diagrams

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- As basic tools for qualitative modelling of uncertainty use probabilistic influence diagrams a.k.a. causal networks, Bayesian networks, Belief networks,.... See the excellent

http://en.wikipedia.org/wiki/Bayesian_network

They are **influence diagrams** with chance nodes only. Qualitatively they describe a probabilistic model through

$$P(A1, A2, \ldots, An) = P(A1 \mid ant(A1)) \ldots P(An \mid ant(An))$$

where ant (Ai) are the antecessors of node Ai.

In what follows we see several PIDs and we need to indicate the entailed probabilistic model

Probabilistic diagrams with two nodes

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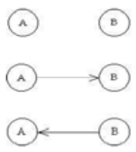

Before moving foreward, write the entailed probabilistic model

Probabilistic diagrams with two nodes

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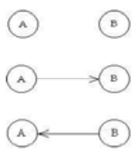
Model for P(A,B)



P(A)P(B)

P(A) P(B|A)

P(B) P(A|B)

First case, A and B are independent. We move from second to third, and viceversa, via Bayes formula

Probabilistic diagrams with three nodes

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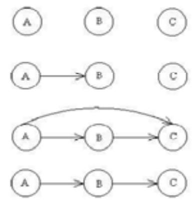

Before moving foreward, write the entailed probabilistic model

Probabilistic diagrams with three nodes

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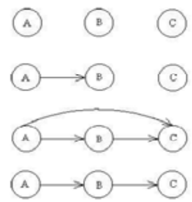
Model P(A ,B ,C)



P(A)P(B)P(C)

P(A) P(B|A) P(C)

P(A)P(B|A)P(C|A,B)

P(A)P(B|A)P(C|B)

First case, independence. Third case, A and C are conditionally independent given B.

Read http://en.wikipedia.org/wiki/Conditional_independence

Probabilistic diagrams. Asia

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We end up with an example referring to lung diseases, from Lauritzen and Spiegelhalter (1988):

A breathing condition (dyspnea) may be due to tuberculosis, lung cancer or bronchitis, none of them or sevral of them. A recent visit to Asia, increase the chances of tuberculosis, whereas smoking is a risk factor for lung cancer and bronchitis. The results of an X-ray may not discriminate between cancer and tuberculosis, as neither the presence or absence of dyspnea does.

Probabilistic diagrams. Asia

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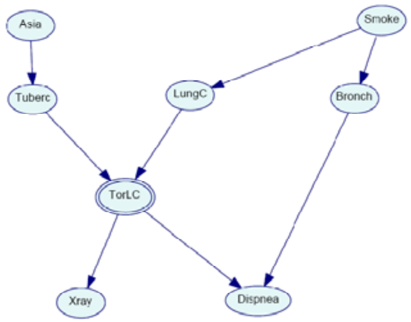
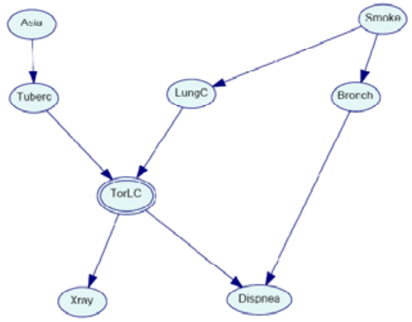


Probabilistic diagrams. Asia

DEIO



P(A,T,S,L,B,O,X,D) = P(A)P(T|A)P(S)P(L|S)P(B|S)P(0|T,L)P(X|O)P(D|O,B)

Eliciting probabilities

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Once the graphical model is built, we must elicit the probabilities. Sometimes we have access to good databases and may approximate probabilities based on relative frequencies. If not, we may use expert judgements...

Eliciting probabilities. The reference experiment

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- A 'ruler' to measure beliefs called reference experiment
- An experiment is a reference experiment for somebody if this person finds all the experiment results equally likely. For me, some reference experiments are:
  - A bag with six identical balls numbered 1,2,...,6. This allows me to measure probabilities with values in between 0, 1/6, 2/6, 3/6,4/6,5/6,6/6=1
  - Throw four balanced coins. This allows me to measure probabilities in between 0, 1/16, 2/16,...., 15/16, 16/16=1
  - A fortune wheel with 14 equal sectors, to measure probabilities in between 0, 1/14, 2/14,...., 14/14=1.

Eliciting probabilities. Protocol

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- Once identified the calibration experiment, we use it to calibrate the probability of the event of interest. The idea is to gradually compare the event of interest with the reference event until we find one which is as likely. This is not easy to do for a beginner, but we may appeal to seevral protocols. One is available at

http://www-math.bgsu.edu/~albert/m115/probability/calibration.html

Eliciting probabilities. Biases

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- At tonite's party, I intorduce you Roman, a shy guy. Is he a salesman or a librarian?

Eliciting probabilities. Biases

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- Salespersons tend to be extrovert, say 9 out of 10 are extrovert.
- However, there are many timid librarians, say 5 out of 10 are.
- But, there are much more salespersons than librarians, say 10 salespersons per librarian.
- Those who said that Roman was a librarian are ignoring such fact.
- More on this by Simon French!!!

Updating probabilities

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- Many times, we additionally have access to evidence (data) which provides information about the event of interest. Our beliefs are updated through Bayes formula

  P(Event|Data)=P(Data|Event) P(Event)/ P(Data)

  More on Bayes formula in
  http://en.wikipedia.org/wiki/Bayes'_theorem
  and in the paper Bayesian methods in conservation biology (just the first four pages)

An example: Unintended slide deployment

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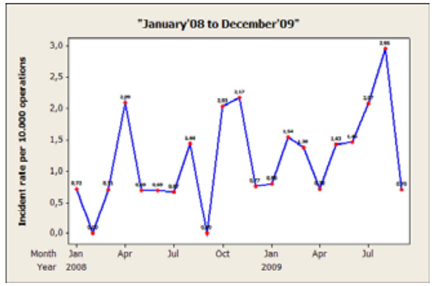

An example: Unintended slide deployment

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An example: Unintended slide deployment

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- Factors affecting incidents
- **Severity analysis (cost)**
- Risk assessment
- Countermeasures?
- Best countermeasure: risk management

- Estimated annual savings 600000 €

An example: Unintended slide deployment

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- Cost
  - Maintenance cost
  - Transportation cost
  - Repair cost
  - Ground delay associated costs

An example: Unintended slide deployment

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- Maintenance Cost
  - Lab x T_m
  - $T_{m.}$ Expert assesses min (30), max (60), most likely (45). Adjust triangular distribution with 0.05, 0.95 quantiles at min, max . Tri (0.385,0.75,1.115)
- Transportation cost

An example: Unintended slide deployment

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- Maintenance cost

$$C_m = q \times C_m^i + (1-q) \times C_m^e$$

- q assumed Beta (16,4)
- C_m

	Bf	Ba	Bw	B2/3
Int. costs	1840	1480	1630	1430
Ext. costs	2605	2323	4571	4741

	A1	A2	A3	A6	A6w
Int. costs	4160	4040	2400	3630	3210
Ext. costs	6429	4850	5785	7423	4946

	Bf	Ba	Bw	B2/3
Incidents	17	4	1	5
Parameters	18	5	2	6

	A1	A2	A3	A6	A6w
Incidents	4	2	1	0	0
Parameters	5	3	2	1	1

An example: Unintended slide deployment

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- Costs in relation with delays

$$T_d = p_0\, I_0 + p_1\, F_d$$

$$p_0 + p_1 = 1$$

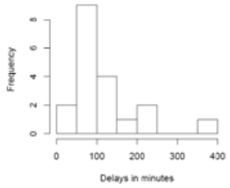
$$p_0, p_1 \geq 0$$

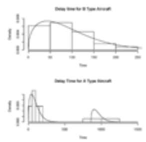
$$p_0 | data \sim Be(14, 23)$$

$$F_{d_B} \sim Wei(\theta = 0, \alpha, \beta)$$

$$F_{d_A} \sim p\, Wei(\theta = 0, \alpha, \beta) + (1-p)\, Wei(\theta, \alpha, \beta),$$

$$f(x \mid \theta, \alpha, \beta) = \alpha \frac{(x-\theta)^{\alpha-1}}{\beta^\alpha} \exp(-((x-\theta)/\beta)^\alpha)$$





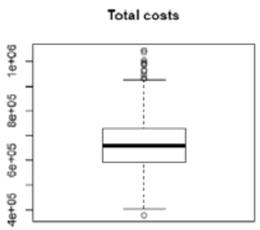
An example: Unintended slide deployment

- Costs in relation with delays

	A Flights	B Flights
	(Min, most likely, max)	(Min, most likely, max)
Passenger Hard Costs	(0.12, 0.19, 0.24)	(0.12, 0.19, 0.24)
Passenger Soft Costs	(0.06, 0.19, 0.22)	(0.06, 0.19, 0.22)
Marginal Crew Costs	(0.00, 14.00, 39.00)	(0.00, 7.90, 16.59)
Marginal Maintenance Costs	(0.65, 0.81, 0.97)	(0.38, 0.47, 0.56)
Total Costs	(0.83, 15.19, 40.27)	(0.56, 8.75, 17.61)

An example: Unintended slide deployment

- Annual costs due to incidents



Sensitivity Analysis

	Has WMD	Does not have WMD
Attack Irak	Save costs due to future attacks	PPRR Disaster
Do not attack Irak	Bigger defense expend.	No change

Consider the following simplified version of the Attack-Irak decision by the US. The main uncertainty is whether Irak has or not WMD. See http://www.lionhrtpub.com/orms/orms-12-06/iraq.html

Sensitivity Analysis

	0.75	0.25	Exp.Util
Attack Irak	50	-70	20
No Attack Irak	-100	0	-75

Consequences evaluated in billions of dollars. We assess that P(Irak WMD)=0.75.

Optimal decision:

Attack

Sensitivity Analysis

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We are unsure about the probability that Irak has WMD. We analyse the impact over the optimal decision. We graph the expected utilities as a function of such probability which we call p.

50 p + (-70) (1-p)
-100 p + 0 (1-p)

And compute the value of p for which both decisions have equal expected utility, which is approx. P=0.32

Sensitivity Analysis

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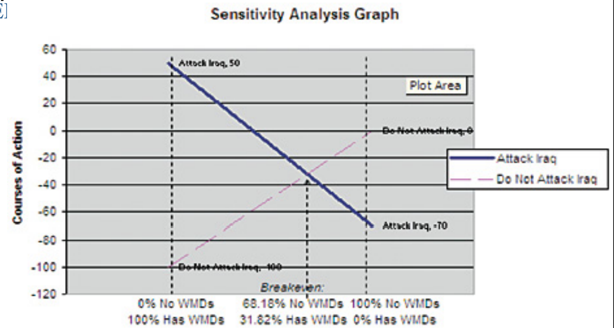

Sensitivity Analysis

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	0.75	0.25	0.32
Attack Irak	50	-70	20
No Attack Irak	-100	0	-75

0.32 is too far from our initial assessment 0.75!!!

Thus, attacking Irak seems uncontroversial...

Scenarios

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In strategic planning, with long range future it may be better to use scenario planning.

Scenarios

- Possible paths into the future
- A few
- Illustrative, distinct, sufficiently complete
- Sensitivity analysis, rather than formal probabilities
- Robust solutions across scenarios